

General Instructions:

1. This Question paper contains – five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 2 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based / case based / passage based / integrated units of assessment (4 marks each) with sub parts.

Section - A [One mark]

1. Let L denotes the set of all straight lines in a plane. Let a relation R be defined by lRm if and only if l is perpendicular to $m \forall l, m \in L$. Then R is:

(a) reflexive ☒ (b) symmetric (c) transitive (d) None of these

2. $A = \{x \in \mathbb{R}, -1 \leq x \leq 1\} = B$. The function $f : A \rightarrow B$ be given by $f(x) = x|x|$ is:

(a) injective but not surjective (b) surjective but not injective

☒ (c) bijective (d) none of these

3. Which of the following functions from \mathbb{Z} into \mathbb{Z} is bijection?

(a) $f(x) = x^3$ ☒ (b) $f(x) = x + 2$ (c) $f(x) = 2x + 1$ (d) $f(x) = x^2 + 1$

4. The value of the expression $2 \sec^{-1} 2 + \sin^{-1} \left(\frac{1}{2} \right)$ is:

(a) $\frac{\pi}{6}$ ☒ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) 1

5. Which of the following is the principal value branch of $\cos^{-1} x$?

(a) $\left[\frac{-\pi}{2}, \frac{\pi}{2} \right]$ (b) $(0, \pi)$ ☒ (c) $[0, \pi]$ (d) $(0, \pi) - \left\{ \frac{\pi}{2} \right\}$

6. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x and y is:

- (a) $x=3, y=1$ ☒ (b) $x=2, y=3$ (c) $x=2, y=4$ (d) $x=3, y=3$

7. A matrix $A = [a_{ij}]_{3 \times 3}$ is defined by $a_{ij} = \begin{cases} 2i+3j, & i < j \\ 5, & i = j \\ 3i-2j, & i > j \end{cases}$

The number of elements in A which are more than 5 is:

- (a) 3 ☒ (b) 4 (c) 5 (d) 6

8. If $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$, then

- (a) Only AB is defined (b) only BA is defined
☒ (c) AB and BA both are defined (d) AB and BA both are not defined

9. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A'$, then:

- (a) $x=0, y=5$ (b) $x+y=5$ ☒ (c) $x=y$ (d) none of these

10. If C_{ij} denotes the cofactor of element P_{ij} of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$, then the value of $C_{31} \cdot C_{23}$ is:

- ☒ (a) 5 (b) 24 (c) -24 (d) -5

11. The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$ is continuous at $x=0$, then the value of k is:

- (a) 3 ☒ (b) 2 (c) 1 (d) 1.5

12. If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is:

- (a) $\frac{1}{2}$ (b) x (c) $\frac{1-x^2}{1+x^2} \{4, -4\}, \phi$ ☒ (d) 1

13. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to:

- (a) $\frac{\cos x}{2y-1}$ (b) $\frac{\cos x}{1-2y}$ (c) $\frac{\sin x}{1-2y}$ (d) $\frac{\sin x}{2y-1}$

14. $y = x(x-3)^2$ decreases for the values of x given by

- (a) $1 < x < 3$ (b) $x < 0$ (c) $x > 0$ (d) $0 < x < \frac{3}{2}$

15. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval

- (a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, 2)$ (c) $(-\infty, 2] \cup [3, \infty)$ (d) $[3, \infty)$

16. The total revenue in rupees received from the sale of x units of a product is given by

$R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$ is:

- (a) 116 (b) 96 (c) 90 (d) 126

17. The rate of change of the area of a circle with respect to its radius r at $r = 6$ cm is:

- (a) 10π (b) 12π (c) 8π (d) 11π

18. The global minimum value of $f(x) = x^4 - x^2 - 2x + 6$ is:

- (a) 6 (b) 8 (c) 4 (d) does not exist

ASSERTION – REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

19. Assertion (A): For a matrix $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ $A \cdot \text{adj } A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

Reason (R): For a square matrix A , $A(\text{adj } A) = (\text{adj } A)A = |A|I$

20. Assertion (A): If $xy = e^{x-y}$ then $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$

Reason (R): $\frac{d}{dx}(U.V) = U \frac{dv}{dx} + V \frac{du}{dx}$

Section - B [Two marks]

- ✓ 21. Check if the relation R in the set $A = \{1, 2, 3, 4\}$ defined as $R = \{(a, b) : a \text{ divides } b\}$ is
(i) symmetric (ii) transitive.

✓ 22. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b .

- ✓ 23. Determine the value of ' k ' for which the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

- ✓ 24. The volume of a cube is increasing at the rate of $9\text{cm}^3/\text{s}$. How fast is its surface area increasing when the length of an edge is 10 cm?
- ✓ 25. Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is strictly increasing.

Section - C [Three marks]

- ✓ 26. Show that the function f in $A = \mathbb{R} - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto.

✓ 27. Prove that $\sin^{-1}\left(\frac{8}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \cot^{-1}\frac{36}{77}$

✓ 28. If $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ find $(AB)^{-1}$.

✓ 29. If $y^x = e^{y-x}$, then prove that $\frac{(1+\log y)^2}{\log y} = \frac{dy}{dx}$.

30. Show that $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of x throughout its domain.

31. Find the local maxima and local minima, of the function $f(x) = \sin x - \cos x$, $0 < x < 2\pi$, also find the local maximum and local minimum values.

Section - D [Five marks]

32. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations:

$$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.$$

33. Discuss the continuity and differentiability of the function:

$$f(x) = |x| + |x-1| \text{ in the interval } (-1, 2).$$

34. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, $0 < t < \frac{\pi}{2}$, find $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

35. Prove that: $\tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$, $0 < x < 1$

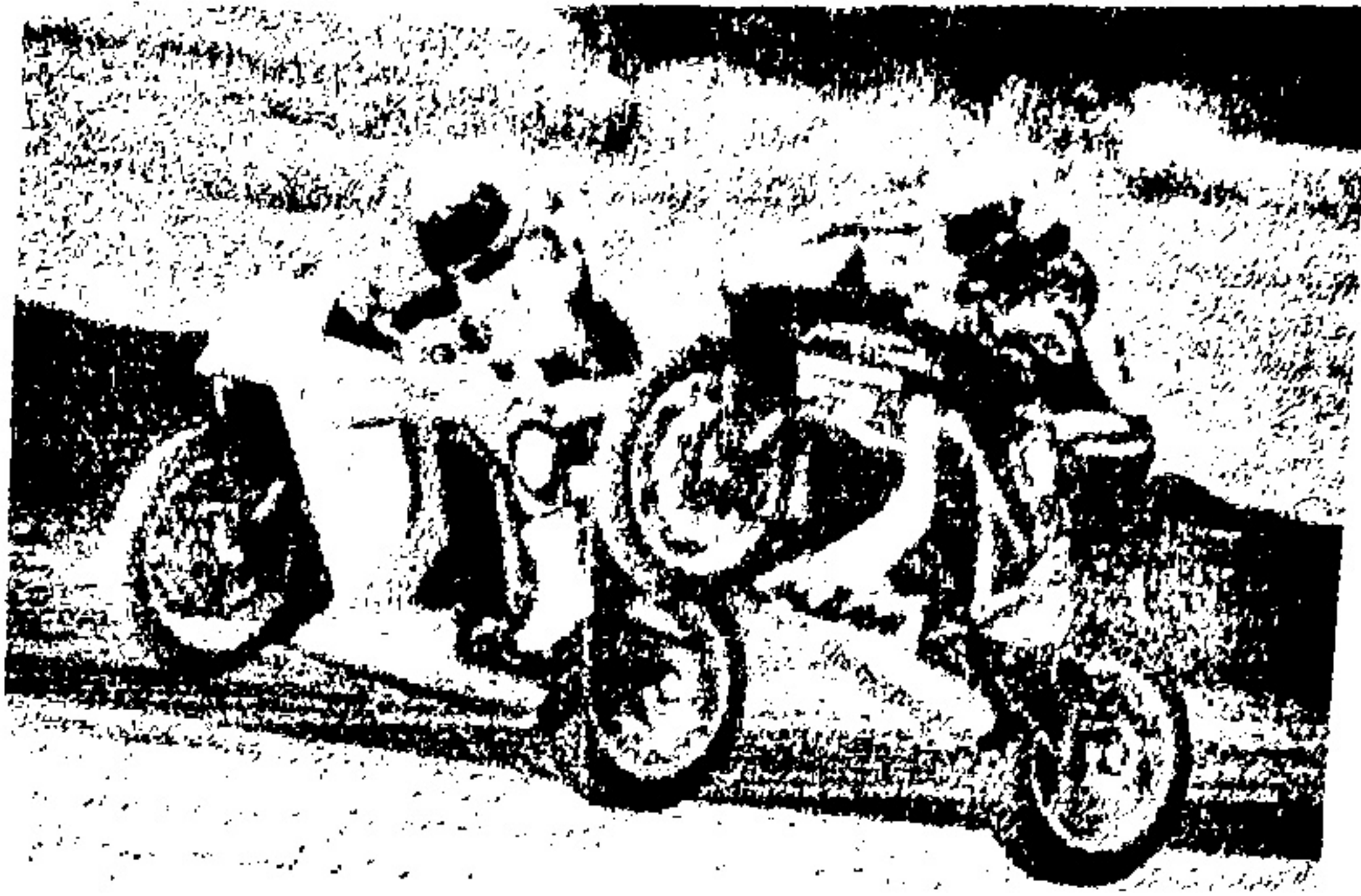
Section - E [Four marks]

Case Study Questions

36. Read the following and answer the following questions:

An organization conducted bike race under 2 different categories – boys and girls. In all, there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race.



Ravi decides to explore these sets for various types of relations and functions:

- (i) Ravi wishes to form all the relations possible from B to G . How many such relations are possible? 2^6 (1)
- (ii) Let $R : B \rightarrow B$ be defined by $R = \{(x, y) : x \text{ and } y \text{ are students of same sex}\}$, describe this relation (equivalence or not). (1)
- (iii) Let $R : B \rightarrow G$ be defined by $R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$ check whether it is bijective or not? And give reason. (2)

37. Read the following and answer the following questions:

Three schools DPS, CVC and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each respectively. The numbers of articles sold are given as:



School / Article	DPS	CVC	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

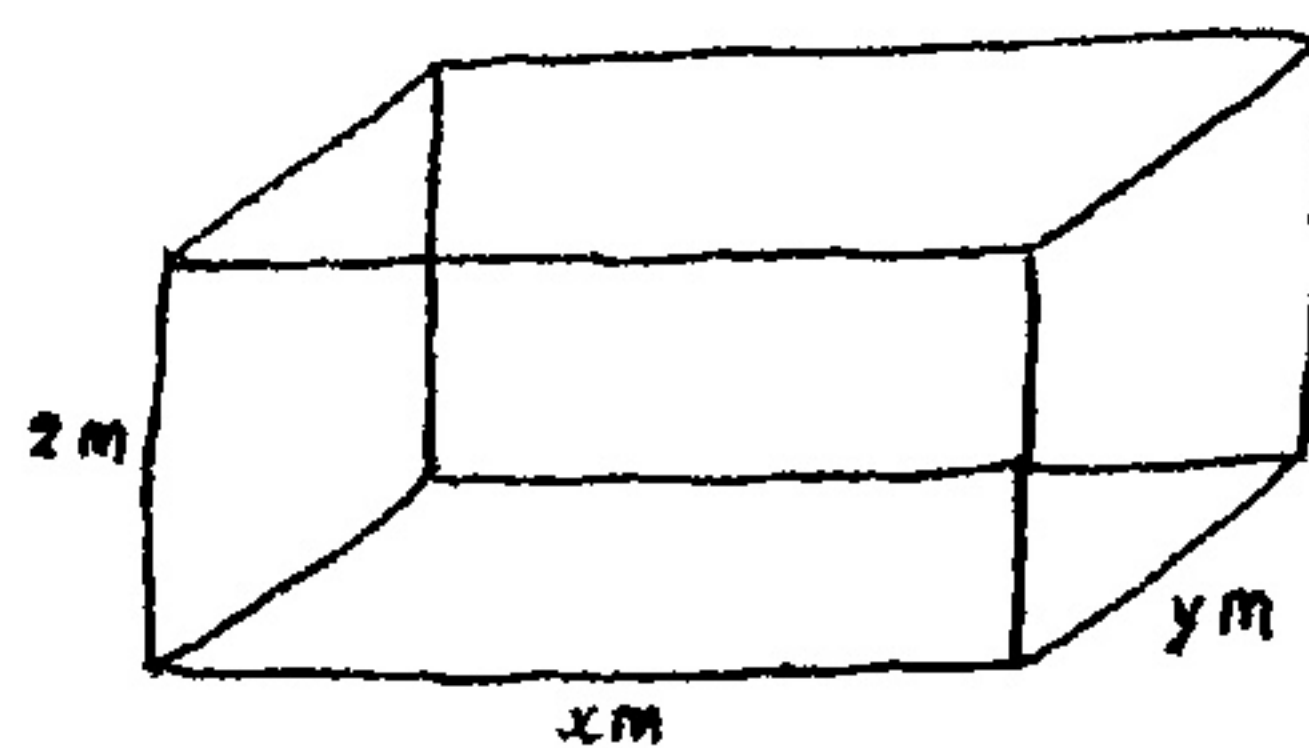
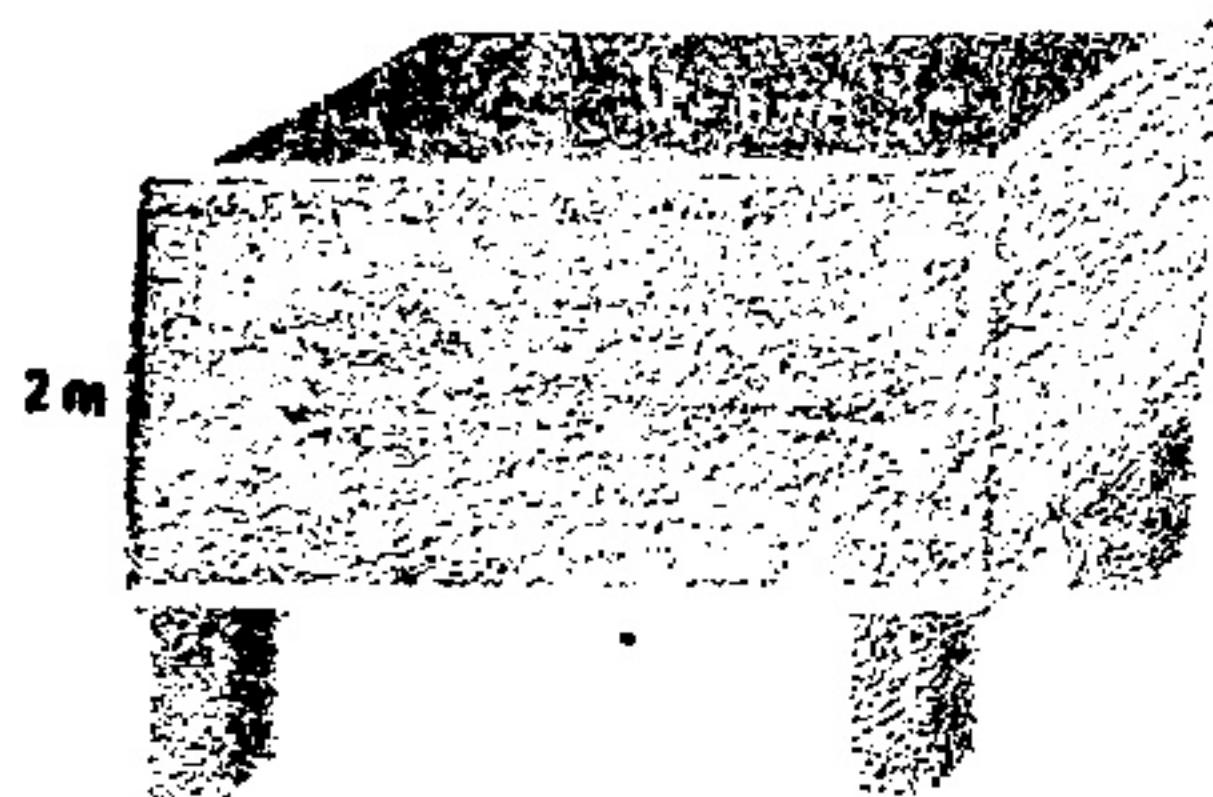
Handwritten calculation:

$$\begin{array}{r} 11500 \\ 6125 \\ \hline 54500 \end{array}$$

- (i) What is the total money (in ₹) collected by the school DPS? (1)
- (ii) What is the total amount of money (in ₹) collected by schools CVC and KVS? (1)
- (iii) If the number of handmade fans and plates are interchanged for all the school, then what is the total money collected by all schools? (2)

38. Read the following and answer the following questions:

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown below:



Based on the above information, answer the following questions:

- (i) If construction of tank cost ₹ 70 per sq. metre for the base and ₹ 45 per square metre for sides, write the making cost 'C' expressed as a function of x . (2)
- (ii) The owner of a construction agency is interested in minimizing the cost 'C' of whole tank, for this to happen, what should be the value of x ? (2)

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$$\frac{x}{7}$$