

CLASS XII

SEPTEMBER 2024

PAPER MATHS

TIME 3 HRS.

M.M.80

SECTION A

(MCQ)

Each question carries 1 marks each

- $\tan^{-1}1 + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) =$
(a) $\frac{3\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $-\frac{3\pi}{4}$
- If A is a matrix of 2×2 and $|A| = 3$, then $|3A| =$
(a) 25 (b) 27 (c) 28 (d) none
- The rate of change of the volume of a sphere with respect to its diameter x is
(a) $32\pi x^3$ (b) $32\pi x^2$ (c) $32\pi x$ (d) none
- When a matrix $\begin{bmatrix} 4 & 1 \\ 0 & x-2 \end{bmatrix}$ is singular then the value of x is
(a) 5 (b) 2 (c) 4 (d) none
- $\int \frac{1}{8x^2+2} dx =$
(a) $\frac{1}{4} \tan^{-1}x + c$ (b) $\frac{1}{4} \tan^{-1}2x + c$ (c) $\tan^{-1}x + c$ (d) none
- The number of all possible matrices of order 2×3 with each entry 1 or 2 are
(a) 16 (b) 32 (c) 64 (d) none
- $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx =$
(a) $\cos \sqrt{x} + c$ (b) $2\cos \sqrt{x} + c$ (c) $2\sin \sqrt{x}$ (d) $\sin \sqrt{x} + c$
- A function $f(x)$ is defined as, $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ now $f(x)$ is
(a) continuous at $x = 0$ (b) not well defined at $x = 0$
(c) discontinuous at $x = 0$ (d) none
- The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval
(a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, 2)$ (c) $(-\infty, 2] \cup [3, \infty)$ (d) $[3, \infty)$
- Let $X = \{x^2: x \in N\}$ and the function $f: N \rightarrow X$ is defined by $f(x) = x^2, x \in N$. Then the function is:
(a) injective (b) not bijective (c) surjective (d) bijective
- If $x = 2 \cos \theta - \cos 2\theta, y = 2 \sin \theta - \sin 2\theta$, then $\frac{dy}{dx}$ is
(a) $\frac{\cos \theta - \cos 2\theta}{\sin 2\theta - \sin \theta}$ (b) $\frac{\cos \theta - \cos 2\theta}{\sin \theta - \sin 2\theta}$ (c) $\frac{\cos 2\theta - \cos \theta}{\sin 2\theta - \sin \theta}$ (d) none

12. The rate of change of the area of a circle with respect to its radius r when $r = 5\text{cm}$.
 (a) $10\pi\text{cm}^2/\text{cm}$ (b) $5\pi\text{cm}^2/\text{cm}$ (c) $20\pi\text{cm}^2/\text{cm}$ (d) none

13. What is the principal value of $(\tan^{-1}\sqrt{3} - \cot^{-1}\sqrt{3})$?

- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) none

14. If A is a square matrix of order 3 and $|A| = -5$, then $|\text{adj } A|$ is

- (a) 25 (b) 125 (c) -25 (d) none

15. If for the matrix $A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α is

- (a) ± 3 (b) 3 (c) -3 (d) none

$$\begin{bmatrix} \alpha-2 & 5 \\ 5 & \alpha-2 \end{bmatrix} \begin{bmatrix} \alpha-2 & \\ -2 & \alpha \end{bmatrix}$$

16. The system of linear equations $5x + ky = 5$, $3x + 3y = 5$ will have unique solution if

- (a) $k \neq 5$ (b) $k = 5$ (c) $k \neq 3$ (d) $k = 3$

17. A function $f: R \rightarrow R$ defined by $f(x) = 2 + x^2$, $x \in R$ is

- (a) One-one (b) onto (c) neither one-one nor onto (d) none

$$2+x^2$$

$$\Rightarrow \begin{bmatrix} \alpha^2+4 & -2\alpha-2\alpha \\ -2\alpha-2\alpha & \alpha^2+4 \end{bmatrix}$$

18. If $\sin y = x \cos(a+y)$, then $\frac{dx}{dy}$ is

- (a) $\frac{\cos a}{\cos^2(a+y)}$ (b) $-\frac{\cos a}{\cos^2(a+y)}$ (c) $\frac{\cos a}{\sin^2 y}$ (d) $-\frac{\cos a}{\sin^2 y}$

$$\rightarrow \cos y = \frac{dx}{dy} (\cos(a+y))$$

$$- \sin(a+y) \frac{dx}{dy} = 1$$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

19. **Assertion (A)** : The domain of the function $\sec^{-1} 2x$ is $(-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$

Reason (R) : $\sec^{-1}(-2) = -\frac{\pi}{4}$

20. **Assertion (A)** : If A is a square matrix such that $A^2 = A$, then $(A + I)^2 - 3A = I$

Reason (R) : $AI = IA = A$

SECTION B

Question from 21 to 25 carries 2 marks each.

21. Show that $f(x) = \cos^2 x$ is a decreasing function on $(0, \frac{\pi}{2})$.

22. Integrate $\frac{1}{1-\cos x}$ w.r.t. x .

23. Evaluate $\int \frac{1}{e^x+1} dx$

24. Differentiate $\tan^2 x$ w.r.t. x .

Find x, y, a, b if $\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$.

SECTION C

Question from 26 to 31 carries 3 marks each.

6. If $x^{13}y^7 = (x+y)^{20}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

7. If $y = [\log(x + \sqrt{x^2 + 1})]^2$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$.

8. Evaluate $\int \frac{x-3}{x^2+2x-4} dx$ OR Evaluate $\int \frac{\cos x}{\sin^2 x + 4 \sin x + 5} dx$. mu

9. Find the intervals in which $f(x) = 2x^3 + 9x^2 + 12x + 20$ is increasing or decreasing.

10. If $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that:

$$I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}.$$

11. Show that the relation R in the set A of all the books in a library of a collage, given by $R = \{(x, y): x \text{ and } y \text{ have the same number of pages}\}$, is an equivalence relation.

SECTION D

Question from 32 to 35 carries 5 marks each.

32. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, find A^{-1} . Using A^{-1} , solve the following system of equation

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

33. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$.
OR

Manufacturer can sell x items at a price of rupees $\left(5 - \frac{x}{100}\right)$ each. The cost price is Rs. $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit.

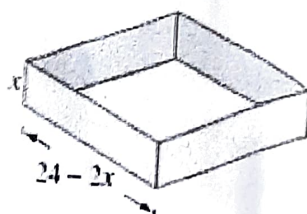
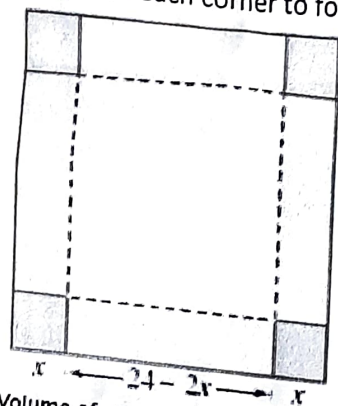
34. Evaluate $\int \frac{x^3}{(x-1)(x-2)(x-3)} dx$.

35. Prove that $y = \frac{4 \sin x}{2 + \cos x} - x$ is an increasing of x in $\left[0, \frac{\pi}{2}\right]$

SECTION E

Case study based questions from 36 to 38. Each sub part of question no. 36 and 37 carries 1,1,2 marks each and sub parts of 38 carries 2,2 marks each.

36. A man has an expensive square shape of goldern board of size 24cm is to be made box without top b cutting from each corner to folding up flaps to form a box



i. Volume of open box formed by folding up the flap:

- ☒ a. $4(x^3 - 24x^2 + 144x)$
- b. $4(x^3 - 34x^2 + 244x)$
- c. $x^3 - 24x^2 + 144x$
- d. $4x^3 - 24x^2 + 144x$

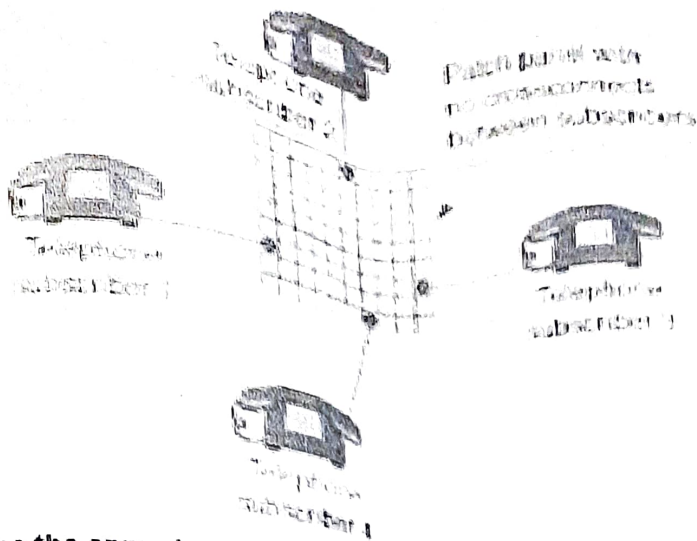
ii. In the first derivative test, if dy/dx changes its sign from positive to negative as x increases through c_1 , then function attain

- a. Local maxima at $x = c_1$
- ☒ b. Local minima at $x = c_1$
- c. Neither maxima nor minima at $x = c_1$
- d. None of these

iii. What should be the side of the square piece to be cut from each corner of the board to be hold the maximum volume

- a. 14 cm
- ☒ b. 12 cm
- c. 4 cm
- d. 5 cm

37. A telephone company in a town has 500 suscribers on its lists and collect fixed xcharges of Rs. 300 per subscriber. The company proposes to increase the annual subscription and it is believed that for every increase of one subscriber will discontinue the service



i. If x be the annual subscription then the total revenue of the company after increment will be:

- a. $R(x) = -x^2 + 200x + 150000$
- b. $R(x) = x^2 - 200x - 140000$
- c. $R(x) = 200x^2 + x + 150000$
- d. $R(x) = -x^2 + 100x + 100000$

ii. To find maximum profit we put

- a. $R'(x) = 0$
- b. $R'(x) > 0$
- c. $R'(x) < 0$
- d. $R''(x) = 0$

iii. How much fee the company should increase to have maximum profit?

- a. 150
- b. Rs. 100
- c. Rs. 200
- d. Rs. 250

38. Three shopkeeper Ram Lal, Shyam Lal and Ghansham Lal are using polythene bags, hand made bags (prepared by prisoner's) and newspaper envelop as carry bags. It is found that the shopkeepers Ram Lal, Shyam Lal and Ghansham Lal are using (20,30,40), (30,40,20), (40,20,30) polythene bags, handmade bags and newspapers envelopes respectively. The three shopkeepers spent RS. 250, RS. 270 and RS. 200 on these carry bags respectively:

i What is the cost of one polythene bag?

- a. Rs. 1
- b. RS. 2
- c. RS. 3
- d. RS. 5

$$20x + 30y +$$

ii What is the cost of one handmade bag?

- a. Rs. 1
- b. RS. 2
- c. RS. 3
- d. RS. 5

$$20x + 30y +$$